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# "…AND NOW PULL IT" – SOLVING CONSTRUCTION PROBLEMS IN STUDENT-FRIENDLY MANNER

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The problems of construction with straightedge and compass are apparently the most ancient and the most important tasks of Euclidean geometry and mathematics. Let us remind the definition of the rational number in Euclid's "Elements" involves the possibility of dividing a unit into arbitrary number of parts with a ruler and compass. For centuries, the search for elegant and effective ways of solutions (including the investigation of possibility of those constructions) was a motive force for development of geometry and mathematics.

Also in school education, problems of construction with 'classic' instruments was a part of secondary- and high-school curricula in most countries. Over their connection with the school subject of technical drawing, they provided a fine pattern for the process of problem solving. However, with the integration of issues of calculus and analytical geometry in high-school curricula, these topics became 'too difficult' and 'less important' - and as a result disappeared from mathematics lessons.

Last years, the wide use of technologies and interactive geometry software (IGS) brings the geometric constructions back to textbooks. The features of the IGS allow for multiple methods of construction (detailed exploring of classical construction problems with IGS is presented in [1]), but also invite a series of didactic dilemma such as using of built-in functions, 'proper and improper' tools for solving problems and so on.

The dynamic nature of IGS provides an opportunity to solve construction problems in the way that simulates the typical reasoning of student. Usually, each construction problem includes more than one requirements, and students have a natural difficulty to build an object that satisfies the whole set of demands. Nevertheless, they do know to solve a problem when only part of requirements imposed.

The leading idea of the proposed didactical approach is the interplay of change and invariance [2]. The problem of construction of the desired object is decomposed into parts and becomes multi-level one. In the initial stage, one needs to construct the geometric object that satisfies some subset of demands. After that, this 'partial solution' is modified in the way that keeps the already satisfied constraints invariant. The appropriate way of modification can be discovered with gradual change of the constructed object, when the trace of 'wrong' points is displayed.

For the wide range of non-trivial construction problems, the construction procedure involves homothetic transformation. This approach provides construction algorithm literally 'by definition': students construct *any* square, rhombus *etc.*, and after that, they *pull* its vertex – up to desirable position, as in Fig. 1.

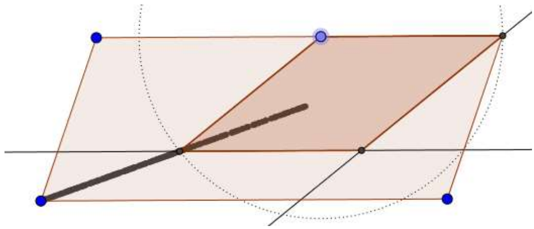

Fig. 1. Construction of a rhombus inscribed in a parallelogram

After discovering the suitable change, the construction itself is straightforward. In cases that are more complicated the tracing helps to discover a hidden invariant and paves the way to the solution (see Fig.2.).

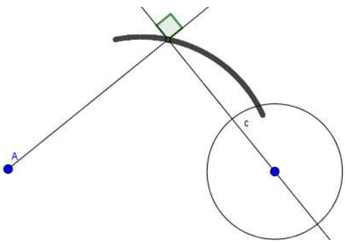


Fig. 2. Construction of a tangent line from a point *A* to the circle *c*

**References:**

[1] Meskens, A., Tytgat, P.: Exploring Classical Greek Construction Problems with Interactive Geometry Software. Springer, Switzerland, 2017.

[2] Sinitsky, I., Ilany, B.: Change and Invariance. A textbook on Algebraic Insight into Numbers and Shapes. Sense Publishers, Rotterdam/Boston/Taipei, 2016.